

JANUARY 19, 2024

## COMPUTER GRAPHICS WS 23/24 ASSIGNMENT 9

The exercises will take place in room G40 in Mühlenpfordtstrasse 23.

This week you will have to do a bit of theoretical work testing what you have learned so far. Complete the assignments and hand in your solutions to these theoretical tasks (with drawings/formulas). Please use different colors in your drawings. Each group hands in one solution. Your group must present the completed assignments on *each Friday, 9:45*.

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### 9.1 Rendering Equation (10 Points)

One form of the rendering equation is:

$$L_o(\mathbf{P}, \vec{\omega}_o) = L_e(\mathbf{P}, \vec{\omega}_o) + \int_{\Omega} f_r(\vec{\omega}_i, \mathbf{P}, \vec{\omega}_o) \cdot \cos\theta_i \cdot L_i(\mathbf{P}, \vec{\omega}_i) \cdot d\omega_i \quad (1)$$

Name and explain the following parameters:

- $\vec{\omega}_o$ :
- $\Omega$ :
- $L_e$ :
- $f_r$ :
- $\theta_i$ :

### 9.2 Convolution theorem (25 Points)

Proof mathematically that a convolution of two functions  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  in the spatial domain equals a multiplication in the Fourier domain, i.e.

$$F\{f * g\} = F\{f\} \cdot F\{g\}$$

### 9.3 Textures (15 Points)

- Explain the MipMap-Concept and compare the storage consumption compared to standard textures.
- Name three ways to create a MipMap from a given Image.
- What is the main difference of anisotropic filtering to standard filtering?

## 9.4 Sampling Formulae (30 Points)

A pixel actually corresponds to a square area. Usually a ray tracer samples the pixels only at their center, which leads to aliasing. To counteract this effect several ray samples are averaged for each pixel. Derive a mathematical formulation for the set of super-sampled rays  $S_r = \{r_{ij} : i, j \in 0, \dots, s-1\}$ , using a base ray  $\vec{r} = \vec{o} + t \cdot \vec{d}$  for each pixel and a function  $\xi = \text{rand}([0, 1])$  where necessary. Assume that you have axes  $\vec{x}, \vec{y}, \vec{z}$  available in the local camera coordinate system, where the camera looks along the  $\vec{z}$  axis.

**Regular Sampling:** The Pixel is subdivided into  $n = s \times s$  equally sized regions, all of which are sampled once in the center.

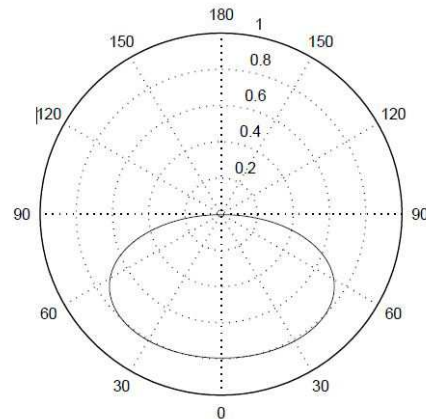
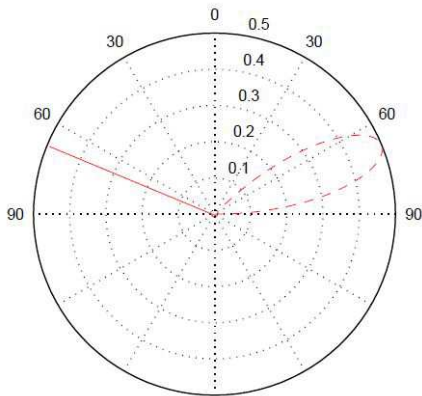
**Random Sampling:** The Pixel is sampled by  $n$  randomly placed samples. All samples must go through the pixel rectangle.

**Stratified Sampling:** Stratified sampling is a combination of regular and random sampling. One sample is randomly placed in each of the  $n = s \times s$  regions.

## 9.5 Two types of goniometric diagrams (7x3 Points)

These two diagrams are typical for describing goniometric properties. The left diagram shows for example a 2d-slice of the reflection properties (dashed curve on the right) of a surface for incident light from an inclination of about  $70^\circ$  (solid line on the left). The right diagram shows typical photometric information for lighting fixtures. The diagram shows a 2d-slice of the intensity-plot of a light source in candelas. The origin of the light source is identical to the center of the plot. Both diagrams are polar plots, i.e. the angle is denoted by rotation around the plot's center, while the intensity is denoted by the distance to the center. The angle values denote the angle between the vertical axis of the light source and a shadow-ray towards the light source. Note the difference in the angle range for the two diagram types.

a) Get acquainted with the two diagram types



b) Sketch (i.e. roughly draw, not up to scale) diagrams for the following reflectance situations: mirror, perfectly diffuse, dusty (higher reflection at grazing angles - think of your old CRT screen), phong (just the specular term)

c) Sketch diagrams for the following light sources: Point light, Spotlight with an opening angle of  $\alpha_{\min} = 20^\circ$  and  $\alpha_{\max} = 40^\circ$  (c.f. to task 3.3) and a linear intensity falloff (the spotlight's direction is collinear with the  $0^\circ$ -line), a point on an area light which illuminates only the positive halfspace of the plane (the normal is collinear with the  $0^\circ$  line).