

DECEMBER 15, 2023

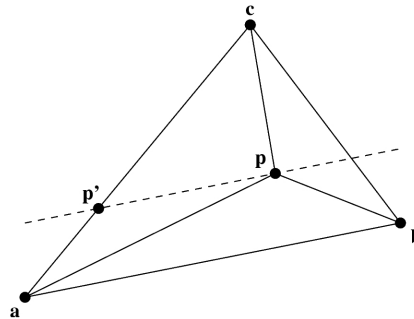
## COMPUTER GRAPHICS WS 23/24 ASSIGNMENT 7

From now on we will do some theoretical work testing and consolidating what you have learned so far. Complete the assignments and hand in your solutions to these theoretical tasks (with drawings/formulas). Please use different colors in your drawings. Each group hands in one solution before **Friday, 9:45**.

---

### 7.1 Barycentric coordinates (24 points)

You have already worked with Barycentric coordinates a lot. We are going to have a closer look at them.



- A plane can also be expressed with the help of an arbitrary point on the plane and two linear independent vectors that span the plane. Give the expression in terms of the vertices.
- Show that, if one Barycentric coordinate remains constant, and the other two vary, the point moves parallel to the side opposite of the constant coordinate.
- Consider the point  $\vec{p}' \in \mathbb{R}^3$  between  $\vec{a}$  and  $\vec{c}$ , defined by the condition that  $(\vec{p} - \vec{p}')$  is parallel to  $(\vec{b} - \vec{a})$ . Show that  $\lambda_3 = \frac{\|\vec{p}' - \vec{a}\|}{\|\vec{c} - \vec{a}\|}$ .
- Show that the Barycentric coordinates can be defined by a quotient of signed areas, for example  $\lambda_3 = \frac{A_{abp}}{A_{abc}}$ .

*Hint:* This exercise is easier, if you do them in order and use the results of each sub-task to guide you to your next solution.

### 7.2 Radiometry (Points 20)

Consider a perfectly spherical, diffuse light bulb with radius 3cm and a radiant power of 60W. Compute the emitted radiance  $L_e(x, \omega_o)$  on the surface of the light bulb.

*Hints:* Diffuse means that the radiance on any surface point and in any possible direction equals the same value  $L$ . To achieve a solution take the formula from the lecture in *differential* form and compute an *integral* form by integration before solving for the unknown.

### 7.3 Phong Model (Points 20 + 10)

Consider the reflection equation

$$L_o(x, \omega_o) = \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos(\theta_i) d\omega_i.$$

- a) The BRDF of the simple Phong model is given by

$$f_r(\omega_i, x, \omega_o) := k_s (\omega_i \cdot \omega_o)^n,$$

where  $0 < k_s < 1$  and  $n > 0$  are constants. Compute the *radiosity*  $\int_{\Omega_+} L_o(x, \omega_o) d\omega_o$  for light coming only from the normal direction, i.e.  $L_i(x, \omega_i) = \delta_n(\omega_i)$  and  $\omega_i \cdot \omega_o = \cos(\theta_o)$ . The *delta distribution*  $\delta_n$  evaluates functions in the normal direction  $n$  with  $\theta_i = 0$ , i.e.

$$\int_{\Omega_+} g(\omega_i) \delta_n(\omega_i) d\omega_i = g(n).$$

**Hint:** Show that  $L_o(x, \omega_i) = k_s \cos(\theta_o)^n$ . The integral over the upper half sphere  $\Omega_+$  can then be reduced to

$$\int_{\Omega_+} L_o(x, \omega_o) d\omega_o = \int_0^{2\pi} \int_0^{\pi/2} k_s \cos(\theta_o)^n \sin(\theta_o) d\theta_o d\phi_o$$

using the relationship  $d\omega_o = \sin(\theta_o) d\theta_o d\phi_o$  for the differential angles. In order to compute the integral, derive the function  $\theta \mapsto (\cos \theta)^m$  using the chain rule.

- b) How would the law of energy conservation for the reflection process look like? Give an informal as well as a mathematical formulation. Does the Phong model satisfy the energy conservation law?

### 7.4 Content comprehension (8 x 2 Points)

These are simple yes or no questions. Note that false answers count **negative** towards your total score!

- a) The Phong reflectance model has a diffuse and a specular term
- b) The Phong reflectance model is energy conserving
- c) Blinn-Phong is preferred over Phong because it is more efficient in terms computation time
- d) Blinn-Phong is preferred over Phong because it looks more nicely
- e) The BRDF depends from wavelengths
- f) The BRDF depends from the viewing angle
- g) The BRDF depends from the distance to the object
- h) The diffuse BRDF is anisotropic

### 7.5 Quantities and Units (10 Points)

Identify the physical quantities *Power*, *Radiant intensity*, *Radiance*, *Luminous flux*, *Luminous intensity*, *Luminance*, *Illuminance* with the units lumen (lm), candela (cd), watt (W), nit, lux (lx), W/sr, W/m<sup>2</sup>/sr. Also match the radiometric quantities with their photometric counterparts and add missing counterparts, if necessary.