The exercises will take place in room G40 in Mühlenpfordtstrasse 23.

This week you will have to do a bit of theoretical work testing what you have learned so far. Complete the assignments and hand in your solutions to these theoretical tasks (with drawings/formulas). Please use different colors in your drawings. Each group hands in one solution. Your group must present the completed assignments on each Friday, 9:45.

Please hand in your solutions at the beginning of the lecture on 18.01.2017, 9:45!

8.1 Sketch kd-tree from point set (30 Points)

In the following you have to sketch a kd-tree for a given (2d) point set. At first, have a look at http://homes.ieu.edu.tr/hakcan/projects/kdtree/kdTree.html to get an overview on kd-tree building for point sets. A typical kd-tree for a point set looks like this:

![Diagram of a kd-tree](image)

In this task we will use the same algorithm for kd-tree building, i.e.: The first axis is vertical. The splitting axis at each level coincides with the median of a given dataset. For a vertical split the left child contains the points left of or on the splitting axis, the right child contains the points right of the splitting axis. For a horizontal split the left child contains the points below the or on the splitting axis, the right child contains the points above the splitting axis. A leaf node contains only one point. Now sketch the kd-trees for the following two point sets:

![Diagram of kd-trees for two point sets](image)

Also provide the tree structure with nodes and links. Name the points $p_1, p_2$, the splitting axis $l_1, l_2$, ...
8.2 Guess the Fourier images (15 Points)

The following frequency domain images (second row) have been created for spatial domain images (first row) using matlab’s \texttt{fft2} method. However, the frequency domain images have been mixed up. Use your knowledge about the transformation properties of edges to guess, which frequency domain image belongs to which spatial domain image.

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8.3 Convolution theorem (25 Points)

Proof mathematically that a convolution of two functions $f(x), g(x)$ in the spatial domain equals a multiplication in the Fourier domain, i.e.

$$F\{f \ast g\} = F\{f\} \cdot F\{g\}$$

8.4 Fourier Transformation (20 Points)

The transformation of a signal $f(x)$ to Fourier space is given by

$$F(\omega) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi i \omega x} dx$$

Consider the box function

$$B_d(x) = \begin{cases} 0 & \text{for } x \leq -d \\ 1 & \text{for } -d < x < d \\ 0 & \text{for } d \leq x \end{cases}$$

and show that its Fourier transformation is a sinc type function. Compare the behaviour of the functions as $x \to \pm \infty$.

8.5 Sampling Formulae (15 Points)

A pixel actually corresponds to a square area. Usually a ray tracer samples the pixels only at their center, which leads to aliasing. Derive valid mathematic formulae for the following cases (let $i$ and $j$ denote the iterators within the pixel area):

Regular Sampling: The Pixel is subdivided into $n = m \times m$ equally sized regions, which are sampled in the middle.

Random Sampling: The Pixel is sampled by $n$ randomly placed samples. The distribution function is chosen arbitrarily and provides sample values $\xi_i \in [0; 1)$. Note, here only one iterator is needed. How would a poisson distributed random value be described mathematically?

Stratified Sampling: Stratified sampling is a combination of regular and random sampling. One sample is randomly placed in each of the $n = m \times m$ regions with $\xi_i, \xi_j \in [0, 1)$. 

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8.6 Textures (15 Points)

a) Explain the MipMap-Concept and compare the storage consumption compared to standard textures.

b) Name three ways to create a MipMap from a given Image.

c) What is the main difference of anisotropic filtering to standard filtering?