Proc. IEEE International Conference on Image Processing (ICIP'03), vol. 3, pp. 717–720, September 2003

SENSITIVITY OF REFLECTED RADIANCE TO SURFACE NORMAL ORIENTATION

Marcus Magnor

Max-Planck-Institut für Informatik
Graphics - Optics - Vision
Stuhlsatzenhausweg 85
D-66123 Saarbrücken, Germany
magnor@mpii.de

ABSTRACT

Given the illumination and the surface normal at a point on a convex surface, the effect of small changes in surface normal direction on observed reflected radiance is examined. A signal-processing framework suitable for the hemispherical geometry of the problem is applied to discuss surface reflection in terms of convolving incident illumination with the reflectance function. Depending on the surface material's reflection properties as well as on the incident illumination distribution, normal direction deviation may or may not be noticeable. Expressions are derived to quantitatively evaluate the amount of change in reflected radiance. Different scenarios exemplify the results. The presented theoretical results have implications for shape-from-shading technqiues as well as for scene-based compression methods.

1. INTRODUCTION

With the ongoing proliferation of digital cameras everywhere, many new application areas for image processing, analysis and interpretation are opening up. In essence, image information retrieval is based on the color and intensity recorded at each pixel. Except for foggy environments and subsurface-scattering materials, pixel color and intensity is determined by the light reflected from the imaged object surface point towards the camera. The appearance of this surface point depends on the incident illumination at the point, the object material's reflection properties, and the viewpoint of the observer. This fundamental image formation process constitutes the basis of computer vision and perceptional psycho-physics, and it defines what can or cannot be read out of image data.

In this paper, a signal-theoretical description of the image formation process is employed to investigate the influence of surface normal orientation on observed reflected radiance. In order to investigate general illumination conditions and arbitrary reflectance functions, for which no analytic expressions may be available, the following discussion is based on a mathematical framework developed in [1]. This framework has previously been applied to examine inverse rendering problems, i.e. the reconstruction of unknown lighting and/or reflection properties from calibrated images when 3D object geometry is known [2]. Related to this work, the influence of surface displacement on coding efficiency, yet without taking surface normal deviation into account, is examined in Ref. [3]. Similarly, a quantitative measure for the range of different appearances of an object is derived in Ref. [4].

2. PRELIMINARIES

In the following discussion, the reflection from a point on the surface of a convex object is considered. While the 3D position of the point remains fixed with respect to the illumination field and the viewing direction, the surface normal point may deviate from its original orientation. To avoid self-shadowing, the object may be convex and the hemisphere above the point is completely unoccluded. Finally, the reflection properties of the object material are assumed to be isotropic, so reflectance does not depend on absolute azimuthal surface orientation. For many natural materials,

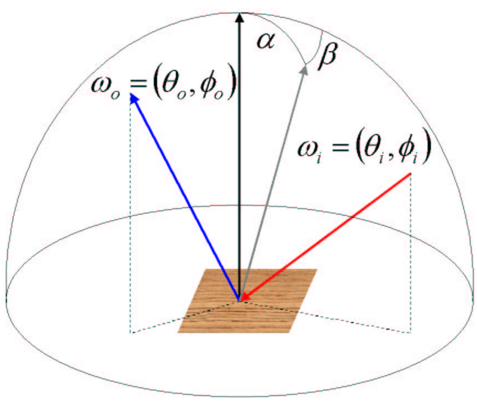

Fig. 1. Reflection geometry and definition of angles.

B	reflected radiance
L	incident/illuminating radiance
ρ	surface reflectance function (BRDF)
$\hat{\rho}$	BRDF times cosine of incident angle
$\omega_i = \theta_i, \phi_i$	incident angle
$\omega_o = \theta_o, \phi_o$	outgoing angle
$\Omega_{i,o}$	sphere of integration
α	deviation of surface normal in elevation
β	deviation of surface normal in azimuth
Y_{lm}	spherical harmonic basis function
l, p	indices of spherical harmonic order
m, q, r	indices of spherical harmonic mode

Fig. 2. Notation.

this is a valid simplification [5]. The used notation is summarized in Fig. 2 and follows Ref. [2].

In Fig. 1, the radiance B reflected into some outgoing direction ω_o depends on the incident illumination L and the reflection properties of the material ρ:

$$B(\omega_o) = \int_{\Omega_i} L(\omega_i)\rho(\omega_i, \omega_o) cos\theta_i \, d\omega_i \quad (1)$$

By definition, the integral may cover the entire sphere Ω_i, while the bi-directional reflectance distribution function (BRDF) ρ is non-zero only above the horizon, i.e. $\rho = 0 \ \forall \ \theta_i < 0, \theta_o < 0$. By incorporating the cosine term into the BRDF function, $\hat{\rho}(\omega_i, \omega_o) = \rho(\omega_i, \omega_o) \cos\theta_i$, the *reflection equation* (1) can be interpreted as a convolution of the incident radiance L and the reflectance function $\hat{\rho}$ (or, mathematically exact, as a scalar product in Hilbert space) [1, 2].

Similar to Fourier expansion in Cartesian coordinates, the constituents of (1) can be expressed in a suitable set of basis functions that are orthonormal in spherical coordinates, in *spherical harmonics* Y_{lm} [6, 7], Fig. 3:

$$L(\theta_i, \phi_i) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} L_{lm} Y_{lm}(\theta_i, \phi_i)$$

$$L_{lm} = \int_{\theta_i=0}^{\pi} \int_{\phi_i=0}^{2\pi} L(\theta_i, \phi_i) Y^*_{lm}(\theta_i, \phi_i) sin\theta_i \, d\theta_i \, d\phi_i$$

and for isotropic BRDFs [2]

$$\hat{\rho}(\theta_i, \theta_o, |\phi_i - \phi_o|) = \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=-\min(l,p)}^{\min(l,p)} \hat{\rho}_{lpq} Y^*_{lq}(\theta_i, \phi_i) Y_{pq}(\theta_o, \phi_o)$$

$$\hat{\rho}_{lpq} = \int_{\Omega_i} \int_{\Omega_o} \hat{\rho}(\omega_i, \omega_o) Y_{lq}(\omega_i) Y^*_{pq}(\omega_o) \times \sin\theta_i \sin\theta_o \, d\omega_i \, d\omega_o \quad (2)$$

such that (1) becomes

$$B(\theta_o, \phi_o) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{p=0}^{\infty} \sum_{q=-\min(l,p)}^{\min(l,p)} L_{lm} \hat{\rho}_{lpq} Y_{pq}(\theta_o, \phi_o). \quad (3)$$

In (1,3), incident illumination $L(\omega_i)$ and surface BRDF $\hat{\rho}(\omega_i, \omega_o)$ are expressed in the same *global* coordinate system. If the surface is tilted against the global (illumination)

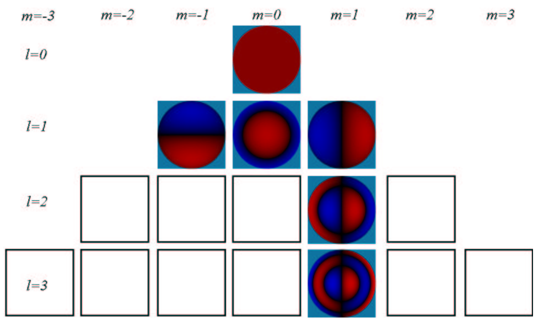

Fig. 3. Spherical harmonics basis functions.

coordinate system by some angles α, β, Fig. 1, $\hat{\rho}$ is defined in the surface's *local* coordinate system.

To evaluate (1) with rotated coordinates for $\hat{\rho}$, incident and outgoing direction must be transformed from the global to the local coordinate system:

$$B(\alpha, \beta, \omega_o) = \int_{\Omega_i} L(\omega_i) \hat{\rho}(R_{\alpha,\beta}(\omega_i), R_{\alpha,\beta}(\omega_o)) \, d\omega_i,$$

where $R_{\alpha,\beta} = R_y(\alpha) R_z(\beta)$ is the rotation operator to transform global to local coordinates. For spherical harmonics, rotating the coordinate system is described by [8, 9]

$$Y_{lm}(R_{\alpha,\beta}(\theta, \phi)) = \sum_{m'=-l}^{l} D^l_{mm'}(\alpha, \beta) Y_{lm'}(\theta, \phi)$$

with

$$D^l_{mm'}(\alpha, \beta) = d^l_{mm'}(\alpha) \ \exp^{I m \beta}$$

$$d^l_{mm'}(\alpha) = N_{l,m,m'} \ \xi^{-(m-m')/2} (1-\xi)^{-(m+m')/2} \times \left(\frac{d}{d\xi}\right)^{l-m} \xi^{l-m'} (1-\xi)^{l+m'}$$

$$N_{l,m,m'} = (-1)^{m-m'} \sqrt{\frac{(l+m)!}{(l-m)!(l+m')!(l-m')!}}$$

$$\xi = \sin^2 \frac{\alpha}{2}. \quad (4)$$

$d^l_{mm'}$ represents a $(2l+1) \times (2l+1)$ matrix. From (4) it is apparent that the rotation mixes only coefficients with different mode index m having the same order l, so rotated spherical harmonic coefficients can be expressed as a linear combination of coefficients of the same order [9].

Expressing the incoming and outgoing direction in global coordinates, (2) reads

$$\hat{\rho}_{\alpha,\beta}(\theta_i, \theta_o, |\phi_i - \phi_o|) = \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} \sum_{q=-\min(l,p)}^{\min(l,p)} \sum_{r=-\min(l,p)}^{\min(l,p)} \hat{\rho}_{lpq} D^l_{qr}(\alpha, \beta) Y^*_{lr}(\theta_i, \phi_i) D^p_{qr}(\alpha, \beta) Y_{pr}(\theta_o, \phi_o)$$

and (3) becomes

$$\begin{aligned} B(\alpha,\beta,\theta_o,\phi_o) &= \sum_{l,m}\sum_{p,q,r} L_{lm}\hat{\rho}_{lpq} D^l_{qr}(\alpha,\beta) D^p_{qr}(\alpha,\beta) Y_{pr}(\theta_o,\phi_o) \\ &= \sum_{p=0}^{\infty}\sum_{r=-p}^{p} B_{pr}(\alpha,\beta) Y_{pr}(\theta_o,\phi_o) \\ B_{pr}(\alpha,\beta) &= \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{\substack{q= \\ -\min(l,p)}}^{\min(l,p)} L_{lm}\ \hat{\rho}_{lpq} \\ & \qquad D^l_{qr}(\alpha,\beta) D^p_{qr}(\alpha,\beta). \end{aligned} \tag{5}$$

With this representation, the effect of varying the normal direction by some angle α, β on reflected radiance B can be investigated.

3. SURFACE NORMAL DECLINATION

Without loss of generality, it can be assumed that the original surface normal is aligned with the global coordinate system ($\alpha, \beta = 0$), yielding $D^{l,p}_{qr} = \delta_{qr}$ in (5). Now, let the surface normal deviate from its original direction by α in elevation and β in azimuth, Fig. 1. The question to be answered is: What is the effect on reflected radiance B along some outgoing direction ω_o ?

From (5), it is apparent that the normal direction affects only D^l_{qr}, D^p_{qr}. For $\alpha = 0$, β has no effect on reflected radiance since the BRDF is isotropic. Changes in reflected radiance can be expected only if $\alpha \neq 0$. Applying the Binomial Theorem and the derivative rule for polynomials to (4), $d^l_{mm'}$ can be rewritten as

$$\begin{aligned} d^l_{mm'} &= N_{lmm'}\ \xi^{-m/2+m'/2}\ (1-\xi)^{-m/2-m'/2} \\ & \left(\sum_{k=m'-m}^{l+m'} \binom{l+m'}{k} (-1)^k \frac{(k+l-m')!}{(k-m'+m)!} \xi^{k-m'+m} \right). \end{aligned}$$

Retaining only first-order terms in ξ yields

$$\begin{aligned} m' = m &: \quad N_{lmm'}\ ((l-m)! + (m(l-m)! - \\ & \qquad \binom{l+m}{1}(l-m+1)!)\ \xi + O(2) \\ m' = m+1 &: \quad -N_{lmm'} \binom{l+m+1}{1}(l-m)!\ \xi^{1/2} + O(1.5) \\ m' = m+2 &: \quad N_{lmm'} \binom{l+m+2}{2}(l-m)!\ \xi + O(2) \\ m' > m+2 &: \quad O(\geq 1.5). \end{aligned}$$

By approximating ξ

$$\xi = \sin^2\alpha/4 = \alpha^2/4 + O(4)$$

the result reads

$$\begin{aligned} d^l_{m=m'} &\approx 1 + N_{lmm}/4\ (m(l-m)! - (l+m)(l-m+1)!)\ \alpha^2 \\ d^l_{m=m'\pm 1} &\approx \pm N_{lmm'}/4\ (l+m+1)(l-m)!\ \alpha \\ d^l_{m=m'\pm 2} &\approx \mp N_{lmm'}/4\ (l+m+2)(l+m+1)/2(l-m)!\ \alpha^2. \end{aligned}$$

A band diagonal matrix with non-zero entries only on the diagonal and its adjacent four secondary diagonals remains. When put into context with (5), it becomes clear that small changes in surface normal direction affect only the matrix elements $D^{l,p}_{qr}(\alpha,\beta)$ with $|q-r| \leq 2$. Thus, changes in reflected radiance $B(\omega_o)$ can be observed if

- the BRDF coefficients $\hat{\rho}_{lpq}$ differ for $-2 \leq q \leq 2$, and
- the observation direction ω_o does not cause $Y_{pr}(\theta_o, \phi_o)$ in (5) to be zero for interesting modes r, and
- the illumination is non-uniform ($L_{lm} \neq 0$ for some $l \neq 0$)

Depending on BRDF properties, variation in reflected radiance may be small, and no information about surface normal orientation may be derivable. On the other hand, by optimizing illumination and observation direction for objects with suitable BRDFs, the accuracy of normal direction estimation may be enhanced.

To observe reflectance changes due to the elevation angle α, the illuminating light field may be symmetric about the surface normal ($L_{lm} = \delta_{m0}$), as long as the illumination is not completely uniform ($L_{l0} \neq 0$ for some $l \neq 0$). To detect reflectance changes due to a varying azimuthal angle β, deviation in elevation angle α must be non-zero, and the illumination symmetry must be broken, so at least some spherical harmonic coefficients $L_{lm\neq 0}$ must be non-zero.

4. EXAMPLES

As shown in the previous section, the detectability of normal direction deviation depends on BRDF properties. In this section, the implications for different BRDF functions are illustrated.

4.1. Mirror Reflection

In the surface point-centered coordinate system, a perfect mirror reflects incoming light into the opposite azimuthal direction but into the same elevation angle:

$$\begin{aligned} \hat{\rho}(\theta_i,\theta_o,|\phi_i-\phi_o|) &= \delta(\cos\theta_o - \cos\theta_i)\delta(\phi_o - \phi_i \pm \pi) \\ &= \int_0^{2\pi}\int_0^{\pi/2} Y^*_{pq}(\theta_i,\phi_i\pm\pi) Y_{lq} \sin\theta_i d\theta_i d\phi_i \\ &= (-1)^q \delta_{lp}. \end{aligned}$$

The BRDF coefficients alternate between -1 and 1 in q. For non-uniform illumination, changes of the normal direction lead to changes in the reflected radiance, as expected. Both the elevation angle α and the azimuthal angle β affect the outgoing radiance. Because higher-order coefficients do not vanish, small changes in normal direction may be detectable if the illumination has equally high frequency components.

4.2. Diffuse Surfaces

For perfectly diffuse (Lambertian) reflectors, the BRDF is a constant corresponding to the surface's albedo. The transfer function $\hat{\rho}$ is comprised of the cosine term only, and the reflected radiance is independent of the outgoing direction

ω_o. Omitting the obsolete index p, the BRDF representation in frequency space reads

$$\begin{aligned}\hat{\rho}(\theta_i) = \max(\cos\theta_i, 0) &= \sum_{l=0}^{\infty}\sum_{q=-l}^{l}\hat{\rho}_{lq}Y_{lq}(\theta_i)\\ \hat{\rho}_{lq} &= \int_{\theta_i=0}^{\pi/2}\int_{\phi_i=0}^{2\pi}Y_{lq}(\theta_i)\cos\theta_i\sin\theta_i d\phi_i d\theta_i\\ \hat{\rho}_{l0} &= 2\pi\int_0^{\pi/2}Y_{l0}(\theta_i)\cos\theta_i\sin\theta_i d\theta_i.\end{aligned}$$

Only the terms with $q = 0$ may be non-zero due to azimuthal independence, so the diagonal elements differ, in general, from the secondary diagonal elements, which are all zero. A deviating normal direction therefore changes the reflected radiance. The coefficient values are [8]

$$\begin{aligned} l = 1 &: \quad \hat{\rho}_{l0} = \sqrt{\frac{\pi}{3}}\\ l > 1,\ odd &: \quad \hat{\rho}_{l0} = 0\\ l\ even &: \quad \hat{\rho}_{l0} = 2\pi\sqrt{\frac{2l+1}{4\pi}}\frac{(-1)^{l/2-1}}{(l+2)(l-1)}\frac{l!}{2^l(l!/2)^2}\end{aligned}$$

which decrease for even orders with $\sim 1/l^2$. For high sensitivity to normal deviations, the illuminating light field must exhibit strong first and second order coefficients $L_{l=1,2}$. Illumination variation at high (even) frequencies, on the other hand, does not improve sensitivity much, while coefficients of odd order > 1 do not have any impact at all [8].

4.3. Dust

Particles smaller than the illuminating wavelength scatter unpolarized light evenly in all directions (Rayleigh scattering). But also macroscopic dust particles and dust-covered surfaces exhibit reflectance behavior that is, to a good approximation, independent of the incoming and the outgoing direction (probably because of the dust particles' fractal surface that is covered by Rayleigh-scattering nano-particles). Examples are TV screens and the Moon. The BRDF is then

$$\begin{aligned}\hat{\rho}(\omega_i, \omega_o) &= \text{const}\\ \hat{\rho}_{lpq} &\neq 0 \text{ only for } l = 0, p = 0 \Rightarrow q = 0\end{aligned}$$

Only the zero-order BRDF coefficient is non-zero. Since multiple modes occur only for higher-order spherical harmonics, changing the normal orientation does not effect the reflected radiance.

5. CONCLUSIONS

The effect of deviating surface normal orientation on reflected radiance depends on surface reflection properties as well as incident illumination. For certain reflectance characteristics or illumination conditions, normal orientation does not influence reflected radiance at all. Dust-covered surfaces exhibit virtually no dependence on normal direction regardless of illumination, and uniform illumination is useless for normal estimation in conjunction with any reflectance property. For other combinations of surface reflectance behavior and illumination, the sensitivity of reflected radiance to normal orientation must be determined individually. The formulae for doing so are presented in this paper.

For shape-from-shading methods, mirroring surfaces are potentially most sensitive to normal deviation, while Lambertian (diffuse) reflection shows only weak dependence on surface normal orientation. Even for suitable reflectance properties, the illuminating light field may or may not allow detecting changes in normal orientation. Symmetric illumination may enable detecting changes in normal elevation angle, while asymmetric illumination is needed to detect deviations in elevation as well as in azimuth. Specularly reflecting materials and asymmetric, high-frequency illumination yield the best setup to estimate normal direction with high accuracy. For compression purposes, on the other hand, the normal direction of diffusely reflecting surfaces need to be encoded only approximately, and the illumination may be represented by the three lowest-order spherical harmonic coefficients.

6. REFERENCES

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